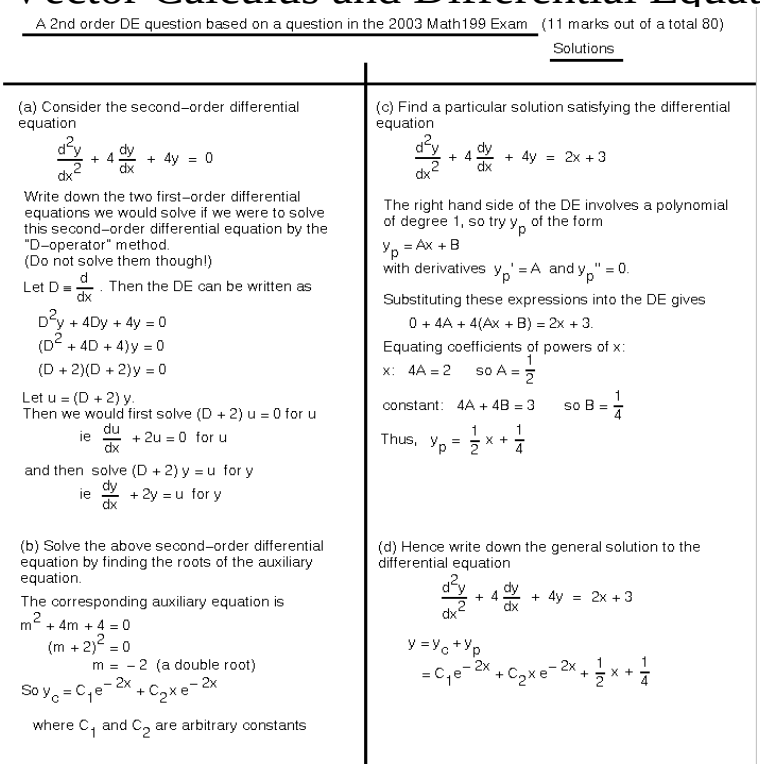
Vector Calculus and Differential Equations



A 2nd order DE question based on a question in the 2003 Math199 Exam (11 marks out of a total 80)

Solutions

(a) Consider the second–order differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

Write down the two first–order differential equations we would solve if we were to solve this second–order differential equation by the "D–operator" method.
(Do not solve them though!)

Let $D \equiv \frac{d}{dx}$. Then the DE can be written as

$D^2y + 4Dy + 4y = 0$

$(D^2 + 4D + 4)y = 0$

$(D + 2)(D + 2)y = 0$

Let $u = (D + 2)\,y$.
Then we would first solve $(D + 2)\,u = 0$ for u

ie $\frac{du}{dx} + 2u = 0$ for u

and then solve $(D + 2)\,y = u$ for y

ie $\frac{dy}{dx} + 2y = u$ for y

(b) Solve the above second–order differential equation by finding the roots of the auxiliary equation.

The corresponding auxiliary equation is

$m^2 + 4m + 4 = 0$

$(m + 2)^2 = 0$

$m = -2$ (a double root)

So $y_c = C_1e^{-2x} + C_2x\,e^{-2x}$

where C_1 and C_2 are arbitrary constants

(c) Find a particular solution satisfying the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 2x + 3$$

The right hand side of the DE involves a polynomial of degree 1, so try y_p of the form

$y_p = Ax + B$

with derivatives $y_p' = A$ and $y_p'' = 0$.

Substituting these expressions into the DE gives

$0 + 4A + 4(Ax + B) = 2x + 3$.

Equating coefficients of powers of x:

x: $4A = 2$ so $A = \frac{1}{2}$

constant: $4A + 4B = 3$ so $B = \frac{1}{4}$

Thus, $y_p = \frac{1}{2}x + \frac{1}{4}$

(d) Hence write down the general solution to the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 2x + 3$$

$y = y_c + y_p$

$= C_1e^{-2x} + C_2x\,e^{-2x} + \frac{1}{2}x + \frac{1}{4}$

This page contains information on the Intermediate Unit of Study MATH Vector Calculus and Differential Equations (advanced). This unit is.This page contains information on the Intermediate Unit of Study MATH Vector Calculus and Differential Equations. This unit is offered in.basic concepts and results in vector integration and vector calculus, Fourier series, and the solution of partial differential equations by separation of variables. fundamental to most areas of applied Maths and many areas of pure Maths, a prerequisite for a large number (11!!) of courses in Semester 2 and in Year 3/4.Differential Equations and Vector Calculus. 6 ECTS credits. - Vector fields, divergence and curl with physical interpretations, potentials, the nabla operator.Course Description. In this course, Krista King from the integralCALC Academy covers a range of topics in Multivariable Calculus, including Vectors, Partial.Review of first-order ordinary differential equations and applications; Higher- order linear differential equations; Laplace Transforms and ODEs. Scalar and vector.Partial Differential Equations and Vector Calculus A students with the fundamental tools required in order to solve simple partial differential equations (PDEs).Vector calculus, or vector analysis, is a branch of mathematics concerned with differentiation Vector calculus plays an important role in differential geometry and in the study of partial differential equations. It is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational.Multivariable Calculus, Linear Algebra, and Differential Equations, Second Edition contains a comprehensive coverage of the study of advanced calculus, linear.Get this from a library! Vector calculus: and differential equations. [Albert G Fadell].Ordinary Differential Equations and Advanced Vector Calculus. A graduate course offered by the Mathematical Sciences Institute. MATH Academic Year.MAT - Multivariate Calculus and Differential Equations () Topics include differential equations, partial differentiation, optimisation, vector calculus.An introductory course in differential equations will contain next to nothing about partial differential equations, so you will be fine to take multivariate calculus and.I would recommend taking multivariable calculus and linear algebra first just to obtain enough basic knowledge to go for the harder differential equations.The specification of curves through differential equations. .. Our subject, multivariable calculus, is concerned with the analysis of functions taking several.